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Orbit method quantization of the AdS_2 superparticle

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Abstract

We consider the Hamiltonian reduction and canonical quantization of a massive AdS_2 superparticle realized on the coset $\text{OSP}(1|2)/\text{SO}(1,1)$. The phase space of the massive superparticle is represented as a coadjoint orbit of a timelike element of $\mathfrak{osp}(1|2)$. This orbit has a well defined symplectic structure and the $\text{OSP}(1|2)$ symmetry is realized as the Poisson bracket algebra of the Noether charges. We then construct canonical coordinates given by one bosonic and one fermionic oscillator, whose quantization leads to the Holstein-Primakoff type realization of $\mathfrak{osp}(1|2)$. We also perform a similar analysis and discuss new features and inconsistencies in the massless case.

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1 Introduction

The quantization of constrained systems is an important problem of modern physics. Certainly a good motivation to be interested in this question is the AdS/CFT correspondence [1], which connects superstring theory in d dimensional Anti-de-Sitter space (AdS_d) to a super conformal field theory (CFT) on the $d-1$ dimensional conformal boundary. For the best studied example, the duality between the type IIB superstring on $\text{AdS}_5 \times \text{S}^5$ [2] and $\mathcal{N} = 4$ super Yang-Mills theory, there has been significant progress during the last decade [3], which can be attributed to existence of integrability in the planar limit [4]. In particular, due to the conjectured quantum integrability, powerful methods have been devised [5], which in principle allow one to predict the spectrum E of arbitrary string states at large 't Hooft coupling, $\lambda = (2\pi R^2 T_0)^2 \gg 1$, see *e.g.* [6]. This ostensibly amounts to quantization of the system.

With all these advances it is worth noting that the quantization of AdS superstrings from first principles is still an open question. For type IIB supergravity on $\text{AdS}_5 \times \text{S}^5$, corresponding to the subsector of half BPS string states, the spectrum has been known for a long time [7]. In [8] it was observed that the reformulation and first quantization of $\text{AdS}_5 \times \text{S}^5$ supergravity in terms of a light-cone scalar superfield [9] is equivalent to quantizing the massless $\text{AdS}_5 \times \text{S}^5$ superparticle. Hence, building on the phase space formalism of [10], the resulting quantization indeed demonstrated a matching of the spectra, see also the related work [11]. Indeed, following the standard literature it seems that a thorough understanding of the massless superparticle would be a useful prerequisite for tackling the superstring.

For massive string states, since the pioneering works [12–15], the majority of study has concentrated on semiclassical string dynamics at large 't Hooft coupling, $\lambda \gg 1$. For these one relies on some of the $\mathfrak{psu}(2,2|4)$ charges diverging as $\sqrt{\lambda}$, rendering the corresponding

string state *long*, or *heavy*, $E \propto \sqrt{\lambda}$. In the BMN limit [12] the total angular momentum on S^5 J diverges. Corrections in $\frac{1}{2P_+} = (E + J)^{-1}$ were computed in [16] and [17], in which the so-called uniform light-cone gauge [18] proved to be convenient. This then led to the perturbative calculation of the scattering S -matrix in this limit [19].

However, in the case of finite $\mathfrak{psu}(2, 2|4)$ charges, for which semiclassical string solutions become *short*, there have been considerable challenges calculating the string spectrum beyond the leading order [13], $E \propto \lambda^{1/4}$. The reason for this is that in this regime the perturbative expansion of the Lagrangian formally breaks down, which can be traced back to the particular scaling behavior of the string zero-modes [20]. For the uniform light cone gauge [18] this is related to $P_- = E - J \propto \lambda^{1/4}$ becoming infinite. This raises the question of whether there is a more useful gauge choice to treat these excitations, or if the computations can be done in a gauge invariant way.

Using static gauge [21] and working in bosonic $\text{AdS}_5 \times S^5$, a generalization of the pulsating string [22] was constructed in [23], which allowed for unconstrained string zero-modes. This so-called single-mode string showed classical integrability and invariance under the isometries $\text{SO}(2, 4) \times \text{SO}(6)$. For the lowest string excitation, dual to a member of the Konishi supermultiplet, the first non-trivial quantum correction to the spectrum was indeed reproduced. The fact that [23] benefited from works on the massive bosonic $\text{AdS} \times S$ particle [24] suggests that for the superstring it would be worthwhile understanding not only the massless, but also the massive $\text{AdS} \times S$ superparticle.

The single-mode string solution of [23] is the $\text{SO}(2, 4) \times \text{SO}(6)$ orbit of the pulsating string [22], motivating the investigation of symmetry group orbits of other semiclassical solutions. This is also appealing as the Kirillov-Kostant-Souriau method of coadjoint orbits, see also the seminal works [25, 26], leads to a quantization in terms of the symmetry generators, which is manifestly gauge-independent. This idea was explored in [27], where, concentrating on the bosonic case of $\text{AdS}_3 \times S^3$, orbits of the particle and spinning string [15] were investigated. In a very natural and succinct way, the quantization procedure gave rise to a Holstein-Primakoff realization for the isometry algebra [28], in agreement with previous results for the particle [24], as well as consistent short and long string limits for the spinning string.

The goal of this work is to generalize the coadjoint orbit method to the case of supergroups. As a first step in this direction, motivated by the AdS/CFT correspondence, we will investigate the AdS_2 superparticle. Recalling that AdS_2 can be realized as the coset $\text{SU}(1, 1)/\text{SO}(1, 1)$, here we will focus on the simplest generalization of this coset, $\text{OSP}(1|2)/\text{SO}(1, 1)$. The superalgebra $\mathfrak{osp}(1|2)$ [29] is the basic, hence classical and simple, Lie superalgebra of lowest dimension [30], and has bosonic subalgebra $\mathfrak{su}(1, 1)$ as required.

Specifying the coset does not determine the action uniquely. Motivated by applications to Green-Schwarz string models, we choose the one possessing κ -symmetry in the massless case. However, in this case the κ -symmetry transformations leave us with an insufficient amount of fermions and consequently the quantization will only be consistent for the massive case.

In fact, non-critical type IIA superstring theory on $\text{OSP}(1|2)/\text{SO}(2)$ has been studied in [31], where due to κ -symmetry and reparametrization invariance, all fermionic and bosonic fields decouple, leaving only a supersymmetric Calogero-Moser model. The related work [32] discussed strings on the coset $\text{OSP}(2|2)/\text{SO}(1,1) \times \text{SO}(2)$, while in [33] the coset model on $\text{OSP}(1|4)/\text{SO}(1,3)$ has been investigated. Furthermore, there have also been works [34] on WZNW models and topological strings for the coset $\text{OSP}(1|2)/\text{SO}(2)$. Apart from these, the present work should also be relevant in the context of gauge/string duality for $\text{AdS}_2 \times \text{S}^2$ [35, 36], see also [37] and the recent works [38].

The paper is organized as follows. After setting up notation in section 2, in section 3 we revise how the coadjoint orbit method works for the massive and massless bosonic AdS_2 particle [26]. We then proceed to the AdS_2 superparticle in section 4, in which we discuss the action of the $\text{OSP}(2|1)/\text{SO}(1,1)$ coset model and its κ -symmetry transformations in the massless case. With this in mind, the coadjoint orbit method is then generalized to the massive and massless AdS_2 superparticle. We conclude and give an outlook in section 5.

2 Notation and Conventions

Using the Pauli matrices σ_j ($j = 1, 2, 3$), a basis of $\mathfrak{su}(1, 1)$ can be written as

$$\mathbf{t}_0 = -i\sigma_3, \quad \mathbf{t}_1 = \sigma_1, \quad \mathbf{t}_2 = \sigma_2. \quad (2.1)$$

The matrices \mathbf{t}_a ($a = 0, 1, 2$) satisfy the relations

$$\mathbf{t}_a \mathbf{t}_b = \eta_{ab} \mathbf{I} + \epsilon_{ab}{}^c \mathbf{t}_c, \quad (2.2)$$

where \mathbf{I} is the unit matrix, $\eta_{ab} = \text{diag}(-1, 1, 1)$ and ϵ_{abc} is the Levi-Civita tensor, with $\epsilon_{012} = 1$. The Killing form defined by the normalized trace $\langle \mathbf{t}_a \mathbf{t}_b \rangle \equiv \frac{1}{2} \text{Tr}(\mathbf{t}_a \mathbf{t}_b) = \eta_{ab}$ provides the isometry between $\mathfrak{su}(1, 1)$ and 3d Minkowski space, since for a pair of $\mathfrak{su}(1, 1)$ vectors $\mathbf{u} = u^a \mathbf{t}_a$ and $\mathbf{v} = v^a \mathbf{t}_a$, one has $\langle \mathbf{u} \mathbf{v} \rangle = u^a v_a$. Expanding $\mathbf{v} \in \mathfrak{su}(1, 1)$ as

$$\mathbf{v} = v_1 \mathbf{t}_1 + v_+ \mathbf{t}_- + v_- \mathbf{t}_+, \quad (2.3)$$

with $\mathbf{t}_\pm = \frac{1}{2}(\mathbf{t}_2 \pm \mathbf{t}_0)$, one finds $v_1 = \langle \mathbf{v} \mathbf{t}_1 \rangle$, $v_\pm = 2\langle \mathbf{v} \mathbf{t}_\pm \rangle$ and

$$\langle \mathbf{v}^2 \rangle = v_1^2 + v_+ v_- . \quad (2.4)$$

We consider the following matrix representation of the real superalgebra $\mathfrak{osp}(1|2)$

$$T_a = \begin{pmatrix} \mathbf{t}_a & 0 \\ 0 & 0 \end{pmatrix}, \quad S_- = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}, \quad S_+ = -i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}, \quad (2.5)$$

where \mathbf{t}_a are given by (2.1). The similarity transformation $U^{-1} T_a U$, $U^{-1} S_\pm U$, with

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i & 0 \\ i & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \quad (2.6)$$

maps (2.5) to the basis vectors of $\mathfrak{osp}(1|2)$ in the usual defining representation.

The commutation relations of the basis elements

$$T_1, \quad T_{\pm} = \frac{1}{2}(T_2 \pm T_0), \quad S_{\pm} \quad (2.7)$$

take the following compact form

$$\begin{aligned} [T_1, T_{\pm}] &= \mp 2T_{\pm}, \quad [T_1, S_{\pm}] = \mp S_{\pm}, \quad [T_{\pm}, S_{\mp}] = S_{\pm}, \quad [T_{\pm}, S_{\pm}] = 0, \\ [T_-, T_+] &= T_1, \quad [S_-, S_+]_+ = -2iT_1, \quad [S_{\pm}, S_{\pm}]_+ = \mp 4iT_{\pm}. \end{aligned} \quad (2.8)$$

The normalized supertrace $\langle \mathfrak{a} \mathfrak{b} \rangle = \frac{1}{2}((\mathfrak{a} \mathfrak{b})_{11} + (\mathfrak{a} \mathfrak{b})_{22} - (\mathfrak{a} \mathfrak{b})_{33})$ defines a Killing form in $\mathfrak{osp}(1|2)$ with nonzero components

$$\langle T_1 T_1 \rangle = 1, \quad \langle T_+ T_- \rangle = \langle T_- T_+ \rangle = \frac{1}{2}, \quad \langle S_+ S_- \rangle = -\langle S_- S_+ \rangle = 2i. \quad (2.9)$$

Then expanding $V \in \mathfrak{osp}(1|2)$ in the basis (2.7)

$$V = V_1 T_1 + V_+ T_- + V_- T_+ + V_+^s S_- + V_-^s S_+, \quad (2.10)$$

one finds $V_1 = \langle V T_1 \rangle$, $V_{\pm} = 2\langle V T_{\pm} \rangle$ and $V_{\pm}^s = \pm \frac{i}{2}\langle V S_{\pm} \rangle$.

3 Coset construction of the bosonic AdS_2 particle

3.1 Classical description

Let us start by considering particle dynamics on AdS_2 , as described by the coset sigma model for $\text{SU}(1,1)/\text{SO}(1,1)$. Explicitly we use the basis for $\text{SU}(1,1)$ given in section 2 and consider the $\text{SO}(1,1)$ gauge transformation $g(\tau) \mapsto e^{\alpha(\tau)\mathbf{t}_1} g(\tau)$ generated by \mathbf{t}_1 . The corresponding gauge invariant action is

$$S = \int d\tau \left[\frac{\langle (\dot{g} g^{-1} - A \mathbf{t}_1)^2 \rangle}{2\xi} - 2\xi \mu^2 \right], \quad (3.1)$$

where ξ is the worldline einbein and A transforms as a gauge potential $A(\tau) \mapsto A(\tau) + \dot{\alpha}(\tau)$. Varying (3.1) with respect to A gives $A = \langle \mathbf{t}_1 \dot{g} g^{-1} \rangle$ and its insertion back in (3.1) leads to the gauge invariant action written solely in terms of g

$$S = \int d\tau \left[\frac{\langle (\dot{g} g^{-1})^2 \rangle - \langle \dot{g} g^{-1} \mathbf{t}_1 \rangle^2}{2\xi} - 2\xi \mu^2 \right]. \quad (3.2)$$

Defining $\mathbf{v} = \dot{g} g^{-1}$ and using (2.3) and (2.4), the action (3.2) can be written as

$$S = \int d\tau \left[\frac{v_+ v_-}{2\xi} - 2\xi \mu^2 \right]. \quad (3.3)$$

The gauge transformation of \mathbf{v} is given by $\mathbf{v} \mapsto e^{\alpha \mathbf{t}_1} \mathbf{v} e^{-\alpha \mathbf{t}_1} + \dot{\alpha} \mathbf{t}_1$. Using $[\mathbf{t}_1, \mathbf{t}_{\pm}] = \mp 2\mathbf{t}_{\pm}$, we obtain $v_{\pm} \mapsto e^{\pm 2\alpha} v_{\pm}$, which explicitly demonstrates the gauge invariance of (3.3).

The action (3.3) can also be written in terms of a Lie algebra valued gauge invariant variable $\mathbf{x} = g^{-1} \mathbf{t}_1 g$. Indeed, one has $\dot{\mathbf{x}} = g^{-1} [\mathbf{t}_1, \mathbf{v}] g$ and $[\mathbf{t}_1, \mathbf{v}] = 2v_+ \mathbf{t}_- - 2v_- \mathbf{t}_+$. Hence, $\langle \dot{\mathbf{x}}^2 \rangle = -4v_+ v_-$ and (3.3) becomes

$$S = \int d\tau \left[-\frac{\dot{x}_a \dot{x}^a}{2\tilde{\xi}} - \frac{\tilde{\xi} \mu^2}{2} \right]. \quad (3.4)$$

Here, $\tilde{\xi} = 4\xi$ and $x_a = \langle \mathbf{x} \mathbf{t}_a \rangle$ are the coordinates of \mathbf{x} in the basis (2.1). These coordinates are real and they are bounded on the hyperboloid $-x_a x^a = (x_0)^2 - (x_1)^2 - (x_2)^2 = -1$, since $\langle \mathbf{x}^2 \rangle = 1$. The time coordinate corresponds to the polar angle in the (x_1, x_2) plane, and hence, after considering the universal cover, the action (3.4) describes the AdS_2 particle with mass μ . This can be seen explicitly by introducing the global coordinates

$$x_1 + ix_2 = \cosh \rho e^{-it}, \quad x_0 = \sinh \rho, \quad g = \exp\left(\frac{\lambda \mathbf{t}_1}{2}\right) \exp\left(\frac{\rho \mathbf{t}_2}{2}\right) \exp\left(\frac{t \mathbf{t}_0}{2}\right), \quad (3.5)$$

such that action (3.4) becomes

$$S = \int d\tau \left[\frac{-\cosh^2 \rho \dot{t}^2 + \dot{\rho}^2}{2\tilde{\xi}} - \frac{\tilde{\xi} \mu^2}{2} \right]. \quad (3.6)$$

The global symmetry of (3.2) is given by the right multiplications $g \mapsto g h$, with $h \in \text{SU}(1, 1)$, and the corresponding Noether charge reads

$$R = \frac{g^{-1} \dot{g} - \langle \dot{g} g^{-1} \mathbf{t}_1 \rangle g^{-1} \mathbf{t}_1 g}{\xi}. \quad (3.7)$$

Writing R in the form

$$R = 2 \begin{pmatrix} -iE & -iB \\ iB^* & iE \end{pmatrix}, \quad (3.8)$$

with $B = B_2 + iB_1$ and $B^* = B_2 - iB_1$, we find that E corresponds to the particle energy, while B_1 and B_2 are the boost generators.

Varying (3.2) with respect to ξ gives the mass-shell condition $\langle R R \rangle + 4\mu^2 = 0$, which is equivalent to the Casimir number relation

$$E^2 - B^* B = \mu^2. \quad (3.9)$$

It is interesting to note that the massive model considered here is classically equivalent to a massless particle moving on $\text{AdS}_2 \times \text{S}^1$ with fixed angular momentum on S^1 . To see this, we extend the $\mu = 0$ case of (3.1) as follows

$$S = \int d\tau \left[\frac{\langle (\dot{g} g^{-1} - A \mathbf{t}_1)^2 \rangle + \dot{\phi}^2}{2\xi} \right], \quad (3.10)$$

where ϕ is the angle on S^1 . Varying with respect to ξ gives the mass-shell condition $\langle (\dot{g} g^{-1} - A \mathbf{t}_1)^2 \rangle + \dot{\phi}^2 = 0$. Furthermore, cyclicity of ϕ yields the integral of motion $\frac{\dot{\phi}}{\xi} = 2\mu$, which when inserted into the mass-shell condition leads us back to (3.9).

3.2 First order formulation and quantization

Applying the Faddeev-Jackiw formalism to (3.2), one finds the first order action

$$S = \int d\tau \left[\langle L \dot{g} g^{-1} \rangle - \frac{\xi}{2} (L_+ L_- + 4\mu^2) - AL_1 \right], \quad (3.11)$$

where L is a Lie algebra valued phase space variable and L_1 , L_{\pm} are its components as in (2.3). The variables ξ and A now play the role of Lagrange multipliers and their variations provide the constraints

$$L_+ L_- + 4\mu^2 = 0, \quad L_1 = 0. \quad (3.12)$$

Thus, the system is described by the 1-form and the Noether charge

$$\Theta = \langle L dg g^{-1} \rangle, \quad R = g^{-1} L g, \quad (3.13)$$

restricted to the constraint surface (3.12). The reduction schemes for the massive and the massless cases are different and hence we analyze them separately.

First we consider the massive case, for which one can use the parametrization $L_{\pm} = \mp 2\mu e^{\pm\gamma}$, for some γ , and hence L can be written as $L = 2\mu e^{\frac{\gamma}{2} \mathbf{t}_1} \mathbf{t}_0 e^{-\frac{\gamma}{2} \mathbf{t}_1}$. Setting $g = e^{\frac{\gamma}{2} \mathbf{t}_1} g_r$, (3.13) then takes a coadjoint orbit form [26]

$$\Theta = 2\mu \langle \mathbf{t}_0 dg_r g_r^{-1} \rangle, \quad R = 2\mu g_r^{-1} \mathbf{t}_0 g_r. \quad (3.14)$$

With the parametrization

$$g_r = e^{\phi \mathbf{t}_0} \begin{pmatrix} \sqrt{1+z^*z} & z \\ z^* & \sqrt{1+z^*z} \end{pmatrix}, \quad (3.15)$$

equation (3.14) reduces to

$$\Theta = \frac{i}{2} (b^* db - b db^*) - 2\mu d\phi, \quad R = 2 \begin{pmatrix} -i(\mu + b^*b) & -i\sqrt{2\mu + b^*b} b \\ ib^* \sqrt{2\mu + b^*b} & i(\mu + b^*b) \end{pmatrix}, \quad (3.16)$$

where $b = \sqrt{2\mu} z$ and $b^* = \sqrt{2\mu} z^*$. From (3.8) we can then extract the Noether charges

$$E = \mu + b^*b, \quad B = \sqrt{2\mu + b^*b} b, \quad B^* = b^* \sqrt{2\mu + b^*b}. \quad (3.17)$$

The symplectic form $\Omega = d\Theta$ obtained from (3.16) is canonical $\Omega = i db^* \wedge db$ and in terms of (B, B^*) it takes the Kirillov-Kostant form

$$\Omega = i \frac{dB^* \wedge dB}{2\sqrt{\mu^2 + B^*B}} = \frac{dB_1 \wedge dB_2}{E}. \quad (3.18)$$

Quantization in the oscillator variables (b, b^*) then leads to the Holstein-Primakoff realization of the symmetry generators (3.17) with Casimir number

$$C = E^2 - \frac{1}{2}(B^*B + BB^*) = \mu(\mu - 1). \quad (3.19)$$

This representation is unitary and irreducible for $\mu > 0$.

Now we consider the massless case. Note that at $\mu = 0$ the 2-form (3.18) is singular at the origin $B_1 = 0 = B_2$. This point corresponds to the massless particle with zero energy and should be removed from the phase space as for Minkowski space. From (3.12) we then have two possibilities, either $L_+ = -e^{\gamma+}$ and $L_- = 0$, or $L_+ = 0$ and $L_- = e^{-\gamma-}$. Let us analyze the second one, which corresponds to $L = e^{\frac{\gamma}{2}\mathbf{t}_1} \mathbf{t}_+ e^{-\frac{\gamma}{2}\mathbf{t}_1}$. As in the massive case, setting $g = e^{\frac{\gamma}{2}\mathbf{t}_1} g_r$ yields

$$\Theta = \langle \mathbf{t}_+ dg_r g_r^{-1} \rangle, \quad R = g_r^{-1} \mathbf{t}_+ g_r, \quad (3.20)$$

and using the parametrization $g_r = e^{2\alpha\mathbf{t}_+} e^{\frac{\beta}{2}\mathbf{t}_1} e^{2\gamma\mathbf{t}_-}$, we obtain

$$\Theta = e^\beta d\gamma, \quad R = e^\beta (\mathbf{t}_+ - 2\gamma\mathbf{t}_1 - 4\gamma^2\mathbf{t}_-). \quad (3.21)$$

The dynamical integrals are then given by

$$E = (\gamma^2 + 1/4)e^\beta, \quad B_1 = -\gamma e^\beta, \quad B_2 = -(\gamma^2 - 1/4)e^\beta, \quad (3.22)$$

and $\Omega = d\Theta = de^\beta \wedge d\gamma$ again takes the Kirillov-Kostant form (3.18) for $\mu = 0$. The case $L_+ = -e^{\gamma+}$ and $L_- = 0$ gives the same answer in a similar way.

The dynamical integrals (3.22) can be expressed through canonical oscillator variables,

$$E = b^*b, \quad B = \sqrt{b^*b} b, \quad B^* = b^* \sqrt{b^*b}. \quad (3.23)$$

with $|b| > 0$, and one arrives again at the Holstein-Primakoff representation for $\mu = 0$. This representation becomes irreducible if one removes the ground state $|0\rangle$, which is annihilated by all symmetry generators. Note that the resulting representation is unitary equivalent to the representation (3.17) for $\mu = 1$.

4 Coset construction of the AdS_2 superparticle

4.1 Classical description

Let us consider $g(\tau) \in \text{OSP}(1|2)$ and the gauge transformations $g(\tau) \mapsto e^{\alpha(\tau)T_1} g(\tau)$. The 'left current' $V = \dot{g} g^{-1}$ then transforms as $V \mapsto e^{\alpha T_1} V e^{-\alpha T_1} + \dot{\alpha} T_1$ and, using the expansion (2.10), we find the following gauge transformations for its components

$$V_1 \mapsto V_1 + \dot{\alpha}, \quad V_\pm \mapsto e^{\pm 2\alpha} V_\pm, \quad V_\pm^s \mapsto e^{\pm \alpha} V_\pm^s. \quad (4.1)$$

We describe a superparticle on AdS_2 by the following gauge invariant action

$$S = \int d\tau \left[\frac{V_+ V_-}{2\xi} - 2\xi \mu^2 \right]. \quad (4.2)$$

The Noether charge related to the right multiplications $g \mapsto g h$ is then given by

$$R = \frac{g^{-1}(V_+ T_- + V_- T_+)g}{\xi}, \quad (4.3)$$

and it satisfies the mass-shell condition $\langle R R \rangle + 4\mu^2 = 0$.

The form of this action is motivated by the supercoset formulation of the Green-Schwarz string on $\text{AdS}_5 \times \text{S}^5$ [2], $\text{AdS}_2 \times \text{S}^2$ [36], AdS_2 with NS-NS [31] or R-R flux [32], and other integrable AdS backgrounds [39]. Removing the dependence of the spacelike worldsheet coordinate the WZ term of those actions drops out and in all cases we are just left with the square of the current projected onto the Grassmann even part of the coset. Furthermore, in the massless case this implies that this action will have a κ -symmetry halving the number of fermionic degrees of freedom. Indeed, in this case the action coincides with a truncation of the $\text{AdS}_2 \times \text{S}^2$ super 0-brane action constructed in [35].

It is also of interest to look at the explicit form of this action. This is readily doable as there are only two Grassmann odd generators in $\mathfrak{osp}(1|2)$ and hence only two fermionic fields. Parametrizing the gauge fixed group field as

$$g = \exp\left(\frac{\psi_1 S_+}{2} + \frac{\psi_2 S_-}{2}\right) \exp\left(\frac{\rho T_2}{2}\right) \exp\left(\frac{t T_0}{2}\right), \quad (4.4)$$

we find the following action

$$S = \int d\tau \left[\frac{(1 + i\psi_1\psi_2)(\cosh \rho \dot{t} + \dot{\rho} - i\psi_1\dot{\psi}_1)(-\cosh \rho \dot{t} + \dot{\rho} + i\psi_2\dot{\psi}_2)}{2\tilde{\xi}} - \frac{\tilde{\xi} \mu^2}{2} \right], \quad (4.5)$$

where $\tilde{\xi} = 4\xi$.

To take the flat space limit we set

$$t \rightarrow \frac{x^0}{R}, \quad \rho \rightarrow \frac{x^1}{R}, \quad \psi_{1,2} \rightarrow \frac{\chi_{1,2}}{\sqrt{R}}, \quad \tilde{\xi} \rightarrow \frac{\tilde{\xi}}{R^2}, \quad \mu \rightarrow \mu R, \quad (4.6)$$

and take $R \rightarrow \infty$. Doing so we find

$$S = \int d\tau \left[\frac{(\dot{x}^0 + \dot{x}^1 - i\chi_1\dot{\chi}_1)(-\dot{x}^0 + \dot{x}^1 + i\chi_2\dot{\chi}_2)}{2\tilde{\xi}} - \frac{\tilde{\xi} \mu^2}{2} \right], \quad (4.7)$$

which can easily be seen to be equivalent to the $2d$, $\mathcal{N} = (1, 1)$ superparticle [40]

$$S = \int d\tau \left[\frac{\eta_{\mu\nu}(\dot{x}^\mu - i\bar{\chi}\gamma^\mu\dot{\chi})(\dot{x}^\nu - i\bar{\chi}\gamma^\nu\dot{\chi})}{2\tilde{\xi}} - \frac{\tilde{\xi} \mu^2}{2} \right]. \quad (4.8)$$

the field content of which is given by two bosons and one Majorana spinor

$$x = \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \quad \eta = \text{diag}(-1, 1), \quad \gamma^0 = \sigma_2, \quad \gamma^1 = i\sigma_1. \quad (4.9)$$

Considering fluctuations around a non-trivial bosonic background, the actions (4.5) and (4.7) describe two bosonic fields satisfying second order equations of motion, and two fermionic fields satisfying first order equations of motion. Therefore, in the massive case, taking account of the mass-shell condition, we find one on-shell bosonic and one on-shell fermionic degree of freedom.

In the massless case, due to the presence κ -symmetry halving the number of fermionic degrees of freedom, we encounter a problem. The κ -symmetry can be most easily seen if we consider the following field redefinitions in (4.5) with $\mu = 0$

$$\begin{aligned}\bar{\xi} &= \text{sech}^2 \rho (1 - i\psi_1 \psi_2) \tilde{\xi} , & x^+ &= t + 2 \tan^{-1} e^\rho , & x^- &= t - 2 \tan^{-1} e^\rho , \\ \chi_1 &= \sqrt{\text{sech} \rho} \psi_1 , & \chi_2 &= \sqrt{\text{sech} \rho} \psi_2 .\end{aligned}\tag{4.10}$$

The resulting action is given by

$$S = \int d\tau \left[-\frac{(\dot{x}^+ - i\chi_1 \dot{\chi}_1)(\dot{x}^- - i\chi_2 \dot{\chi}_2)}{2\bar{\xi}} \right] ,\tag{4.11}$$

which remarkably is formally equivalent to the massless case of the $2d, \mathcal{N} = (1, 1)$ action (4.7) when we take $x^\pm = x^0 \pm x^1$ and $\bar{\xi} = \tilde{\xi}$. The action (4.11) is then invariant under the following κ -symmetry transformation [41]

$$\begin{aligned}\delta \hat{\xi} &= -2i\tilde{\xi}(\dot{\chi}_1 \kappa_2 + \dot{\chi}_2 \kappa_1) , & \delta x^+ &= -i\chi_1 P^+ \kappa_2 , & \delta x^- &= -i\chi_2 P^- \kappa_1 , \\ \delta \chi_1 &= -P^+ \kappa_2 , & \delta \chi_2 &= -P^- \kappa_1 ,\end{aligned}\tag{4.12}$$

where $\kappa_{1,2}$ are infinitesimal Grassmann odd parameters that are allowed to depend on τ and we have defined

$$P^+ = \dot{x}^+ - i\chi_1 \dot{\chi}_1 , \quad P^- = \dot{x}^- - i\chi_2 \dot{\chi}_2 .\tag{4.13}$$

Note that the mass-shell condition following from (4.11) is $P_+ P_- = 0$ and hence the on-shell rank of the κ -symmetry transformations (4.12) is one.

As we will see, this problem will reappear when we try to quantize the massless AdS_2 superparticle based on the action (4.2) and the supergroup $\text{OSP}(1|2)$. Indeed, the fact that we do not have enough fermionic degrees of freedom is a consequence of the fact that we started with the superalgebra $\mathfrak{osp}(1|2)$, which has only two fermionic generators. To properly treat the massless superparticle on AdS_2 we should instead start with the superalgebra $\mathfrak{su}(1, 1|1) \simeq \mathfrak{osp}(2|2)$, gauging an $\mathfrak{so}(1, 1) \oplus \mathfrak{u}(1)$ subalgebra [32], which has twice the number of fermionic generators.

4.2 First order formulation

In the first order formalism (4.2) is equivalent to

$$S = \int d\tau \left[\langle L \dot{g} g^{-1} \rangle - \frac{\xi}{2} (L_+ L_- + 4\mu^2) - A_1 L_1 - A_+^s L_-^s - A_-^s L_+^s \right] ,\tag{4.14}$$

where L_1, L_\pm, L_\pm^s are the components of L in the basis (2.7), (ξ, A_1, A_\pm^s) play the role of Lagrange multipliers and their variations give the constraints

$$L_+ L_- + 4\mu^2 = 0 , \quad L_1 = 0 , \quad L_\pm^s = 0 .\tag{4.15}$$

As in the bosonic case, we have the 1-form $\Theta = \langle L dg g^{-1} \rangle$ and the Noether charge $R = g^{-1} L g$, only now $g \in \text{OSP}(1|2)$, $L \in \mathfrak{osp}(1|2)$ and the system has to be reduced to the

constraint surface (4.15). Similarly to the bosonic case, L and g can then be parameterized $L = 2\mu e^{\frac{\gamma}{2} T_1} T_0 e^{-\frac{\gamma}{2} T_1}$, $g = e^{\frac{\gamma}{2} T_1} g_r$, which leads to

$$\Theta = 2\mu \langle T_0 dg_r g_r^{-1} \rangle, \quad R = 2\mu g_r^{-1} T_0 g_r. \quad (4.16)$$

To find a suitable parametrization of g_r we represent it as $g_r = g_f g_b$, with g_f purely fermionic and g_b purely bosonic. Setting $g_f = e^{\theta_+ S_- + \theta_- S_+}$, where S_{\pm} are the fermionic generators in (2.5) and θ_{\pm} are real Grassmann odd parameters, we find

$$g_f = \begin{pmatrix} 1 + \frac{\theta^* \theta}{2} & 0 & \theta \\ 0 & 1 + \frac{\theta^* \theta}{2} & \theta^* \\ -\theta^* & \theta & 1 - \theta^* \theta \end{pmatrix}, \quad (4.17)$$

with $\theta = \theta_+ - i\theta_-$, $\theta^* = \theta_+ + i\theta_-$. Similarly to (3.15), g_b is chosen as

$$g_b = e^{\phi T_0} \begin{pmatrix} \sqrt{1+u^*u} & u & 0 \\ u^* & \sqrt{1+u^*u} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4.18)$$

and the product of (4.17) and (4.18) can be written as

$$g_r = e^{\phi T_0} \begin{pmatrix} \sqrt{1+z^*z} + \frac{\psi^* \psi}{2\sqrt{1+z^*z}} & z & \psi \\ z^* & \sqrt{1+z^*z} + \frac{\psi^* \psi}{2\sqrt{1+z^*z}} & \psi^* \\ z^* \psi - \sqrt{1+z^*z} \psi^* & \sqrt{1+z^*z} \psi - z \psi^* & 1 - \psi^* \psi \end{pmatrix}, \quad (4.19)$$

where $\psi = \theta e^{i\phi}$, $z = u(1 + \frac{\theta^* \theta}{2})$ and ψ^* , z^* are their complex conjugations.

The inverse to (4.19) is given by

$$g_r^{-1} = \begin{pmatrix} \sqrt{1+z^*z} + \frac{\psi^* \psi}{2\sqrt{1+z^*z}} & -z & z\psi^* - \sqrt{1+z^*z} \psi \\ -z^* & \sqrt{1+z^*z} + \frac{\psi^* \psi}{2\sqrt{1+z^*z}} & z^* \psi - \sqrt{1+z^*z} \psi^* \\ \psi^* & -\psi & 1 - \psi^* \psi \end{pmatrix} e^{-\phi T_0}, \quad (4.20)$$

and then the 1-form and the Noether charge in (4.16) become

$$\Theta = \frac{i}{2}(b^* db - b db^*) + \frac{i}{2}(f^* df + f df^*) - 2\mu d\phi, \quad R = \begin{pmatrix} -2iE & -2iB & -iF \\ 2iB^* & 2iE & iF^* \\ -iF^* & -iF & 0 \end{pmatrix}, \quad (4.21)$$

where $b = \sqrt{2\mu} z$, $b^* = \sqrt{2\mu} z^*$, $f = \sqrt{2\mu} \psi$, $f^* = \sqrt{2\mu} \psi^*$ are canonical coordinates and the matrix elements in (4.21) read

$$\begin{aligned} E &= \mu + b^* b + \frac{f^* f}{2} \\ B &= \sqrt{2\mu + b^* b} b + \frac{f^* f}{2\sqrt{2\mu + b^* b}} b & B^* &= (B)^*, \\ F &= \sqrt{2\mu + b^* b} f + f^* b, & F^* &= (F)^*. \end{aligned} \quad (4.22)$$

The canonical coordinates define the Poisson brackets

$$\{A_1, A_2\} = i \left(\frac{\partial A_1}{\partial b^*} \frac{\partial A_2}{\partial b} - \frac{\partial A_1}{\partial b} \frac{\partial A_2}{\partial b^*} \right) - i \left(\frac{\overleftarrow{\partial} A_1}{\partial f^*} \frac{\overrightarrow{\partial} A_2}{\partial f} + \frac{\overleftarrow{\partial} A_1}{\partial f} \frac{\overrightarrow{\partial} A_2}{\partial f^*} \right), \quad (4.23)$$

where $\overrightarrow{\partial}$ and $\overleftarrow{\partial}$ denote the left and the right derivatives, respectively. The Poisson brackets of the dynamical integrals (4.22) satisfy the $\mathfrak{osp}(1|2)$ algebra

$$\begin{aligned} \{E, B\} &= i B, & \{E, B^*\} &= -i B^*, & \{B, B^*\} &= -2i E, \\ \{E, F\} &= \frac{i}{2} F, & \{B, F\} &= 0, & \{B^*, F\} &= i F^*, \\ \{E, F^*\} &= -\frac{i}{2} F^*, & \{B, F^*\} &= -i F, & \{B^*, F^*\} &= 0, \\ \{F, F^*\} &= -2i E, & \{F, F\} &= -2i B, & \{F^*, F^*\} &= -2i B^*. \end{aligned} \quad (4.24)$$

The Casimir number obtained from (4.22) corresponds to the mass-square

$$C = E^2 - B^* B - \frac{1}{2} F^* F = \mu^2, \quad (4.25)$$

and the energy given in terms of other symmetry generators is

$$E = E_B + \frac{F^* F}{4E_B}, \quad \text{with } E_B = \sqrt{\mu^2 + B^* B}. \quad (4.26)$$

From (4.22) we also find

$$F^* F = 2\mu f^* f, \quad (4.27)$$

which allows us to invert the map from (b, b^*, f, f^*) to (B, B^*, F, F^*)

$$\begin{aligned} b &= \left(1 - \frac{F^* F}{8\mu E_B} \right) \frac{B}{\sqrt{\mu + E_B}}, & b^* &= (b)^*, \\ f &= \frac{\sqrt{\mu + E_B} F}{2\mu} - \frac{B F^*}{2\mu \sqrt{\mu + E_B}}, & f^* &= (f)^*. \end{aligned} \quad (4.28)$$

Using then the coordinates $\xi^k = (B, B^*, F, F^*)$, we can write the canonical 1-form $\Theta = \frac{i}{2}(b^* db - b db^*) + \frac{i}{2}(f^* df + f df^*)$ as follows

$$\Theta = \Theta_B dB + \Theta_{B^*} dB^* + \Theta_F dF + \Theta_{F^*} dF^*,$$

with

$$\begin{aligned} \Theta_B &= \frac{i}{4\mu^2} \left(\frac{2\mu^2}{E_B + \mu} - \frac{F^* F}{2E_B} \right) B^*, & \Theta_{B^*} &= (\Theta_B)^*, \\ \Theta_F &= \frac{i}{4\mu^2} (E_B F^* - B^* F), & \Theta_{F^*} &= -(\Theta_F)^*. \end{aligned} \quad (4.29)$$

The matrix elements for the symplectic form $\Omega = d\Theta$ are given by

$$\begin{aligned} \Omega_{BB} &= \Omega_{B^* B^*} = 0, & \Omega_{BB^*} &= -\Omega_{B^* B} = \partial_B \Theta_{B^*} - \partial_{B^*} \Theta_B, \\ \Omega_{BF} &= \Omega_{FB} = \partial_B \Theta_F + \partial_F \Theta_B, & \Omega_{BF^*} &= \Omega_{F^* B} = \partial_B \Theta_{F^*} + \partial_{F^*} \Theta_B, \\ \Omega_{B^* F} &= \Omega_{FB^*} = \partial_{B^*} \Theta_F + \partial_F \Theta_{B^*}, & \Omega_{B^* F^*} &= \Omega_{F^* B^*} = \partial_{B^*} \Theta_{F^*} + \partial_{F^*} \Theta_{B^*}, \\ \Omega_{FF} &= 2\partial_F \Theta_F, & \Omega_{F^* F^*} &= 2\partial_{F^*} \Theta_{F^*}, & \Omega_{FF^*} &= \Omega_{F^* F} = \partial_F \Theta_{F^*} + \partial_{F^*} \Theta_F, \end{aligned} \quad (4.30)$$

where all derivatives are left derivatives. We then obtain the symplectic matrix

$$\Omega_{kl} = \frac{i}{4\mu^2} \begin{pmatrix} 0 & -A & \frac{B^*F^*}{E_B} & -F^* \\ A & 0 & -F & \frac{BF}{E_B} \\ \frac{B^*F^*}{E_B} & -F & -2B^* & 2E_B \\ -F^* & \frac{BF}{E_B} & 2E_B & -2B \end{pmatrix}, \quad \text{with} \quad A = \frac{4\mu^2 - F^*F}{2E}. \quad (4.31)$$

This equations generalize the symplectic form (3.18) for the $\text{OSP}(1|2)$ coadjoint orbits.

According to (4.24) the matrix formed by the Poisson brackets $\Omega^{kl} = \{\xi^k, \xi^l\}$ reads

$$\Omega^{kl} = \begin{pmatrix} 0 & -2iE & 0 & -iF \\ 2iE & 0 & iF^* & 0 \\ 0 & -iF^* & -2iB & -2iE \\ iF & 0 & -2iE & -2iB^* \end{pmatrix}. \quad (4.32)$$

It is straightforward to check that this matrix inverts the symplectic matrix (4.31), demonstrating the consistency of the calculations.

Now we consider the massless superparticle on AdS_2 . Here, as for the bosonic case (see (3.20)), one should analyze the 1-form and the Noether charge given by

$$\Theta = \langle T_+ dg_r g_r^{-1} \rangle, \quad R = g_r^{-1} T_+ g_r. \quad (4.33)$$

We use the representation $g_r = g_f g_b$, where $g_f = e^{\theta_+ S_- + \theta_- S_+}$, as in (4.17), and g_b is parameterized similarly to the massless bosonic case $g_b = e^{2\alpha T_+} e^{\frac{\beta}{2} T_1} e^{2\gamma T_-}$. The 1-form Θ then splits into a sum of fermionic and bosonic differentials $\Theta = \langle T_+ dg_f g_f^{-1} \rangle + \langle g_f^{-1} T_+ g_f dg_b g_b^{-1} \rangle$.

Using the expansion

$$g_f = I + \theta_+ S_- + \theta_- S_+ + \frac{\theta_+ \theta_-}{2} (S_- S_+ - S_+ S_-) \quad (4.34)$$

and the algebra (2.8), we find

$$g_f^{-1} T_+ g_f = (1 - 2i\theta_+ \theta_-) T_+ + \theta_+ S_+, \quad (4.35)$$

$$dg_f g_f^{-1} = 2i(\theta_+ d\theta_+ T_- - \theta_- d\theta_- T_+) - i(\theta_+ d\theta_- + \theta_- d\theta_+) T_1 + (1 + i\theta_+ \theta_-) (d\theta_+ S_- + d\theta_- S_+).$$

The calculation of the bosonic part is similar to (3.21). Finally we obtain

$$\Theta = i\theta_+ d\theta_+ + e^\beta (1 - 2i\theta_+ \theta_-) d\gamma, \quad (4.36)$$

$$R = e^\beta (1 - 2i\theta_+ \theta_-) (T_+ - 2\gamma T_1 - 4\gamma^2 T_-) + e^{\beta/2} \theta_+ (S_+ - 2\gamma S_-). \quad (4.37)$$

Introducing the new bosonic variable by $e^{\tilde{\beta}} = e^\beta (1 - 2i\theta_+ \theta_-)$ we conclude that the dependence on θ_- drops out, which demonstrates the κ -symmetry discussed above.

4.3 Quantization

Let us introduce the standard bosonic and fermionic creation-annihilation operators (b^\dagger, b) and (f^\dagger, f) , which satisfy the canonical commutation relations $[b, b^\dagger] = 1$ and $\{f, f^\dagger\} = 1$. The operators for the Noether charges are defined on the basis of the classical representation (4.22) and the operator ordering freedom is fixed by

$$\begin{aligned} E &= \mu + b^\dagger b + \frac{f^\dagger f}{2} , \\ B &= \sqrt{2\mu + b^\dagger b + f^\dagger f} b , & B^* &= B^\dagger , \\ F &= \sqrt{2\mu + b^\dagger b + f^\dagger f} f + f^\dagger b , & F^* &= F^\dagger . \end{aligned} \quad (4.38)$$

Note that the classical expressions for B and F in (4.22) can also be written in this form. This form of the symmetry generators becomes helpful for calculating of commutation relations.

The operators (4.38) act in the Hilbert space spanned by the energy eigenvectors $|n, m\rangle$, with $n \geq 0$ and $m = (0, 1)$. The energy spectrum, therefore, is

$$E_{nm} = \mu + n + \frac{m}{2} . \quad (4.39)$$

The action of the operator $\sqrt{2\mu + b^\dagger b + f^\dagger f}$ on the energy eigenstates is defined as in the Holstein-Primakoff representation by

$$\sqrt{2\mu + b^\dagger b + f^\dagger f} |n, m\rangle = \sqrt{2\mu + n + m} |n, m\rangle . \quad (4.40)$$

It is straightforward to check that the operators (4.38) satisfy the commutation relations of the $\mathfrak{osp}(1|2)$ algebra

$$\begin{aligned} [E, B^\pm] &= \pm B^\pm , & [B^-, B^+] &= 2E , \\ [E, F^\pm] &= \pm \frac{1}{2} F^\pm , & [B^\pm, F^\pm] &= 0 , & [B^\pm, F^\mp] &= \mp F^\pm , \\ \{F^+, F^-\} &= 2E , & \{F^\pm, F^\pm\} &= 2B^\pm , \end{aligned} \quad (4.41)$$

where we have defined $B^- = B$, $B^+ = B^*$, $F^- = F$ and $F^+ = F^*$.

The calculation of the quantum Casimir number from (4.38) yields

$$C = E^2 - \frac{1}{2}(B^- B^+ + B^+ B^-) - \frac{1}{4}(F^+ F^- - F^- F^+) = \mu(\mu - 1/2) . \quad (4.42)$$

The massless case corresponds to $\mu = 0$. As in the bosonic case the vacuum is invariant under the action of all symmetry generators. Therefore, to construct an irreducible representation, one has to remove the state $|0, 0\rangle$. One can show that the resulting representation is unitary equivalent to the representation (4.38) at $\mu = 1/2$.

5 Conclusion

In this article we have canonically quantized a massive AdS_2 superparticle on the basis of the superisometry group $\text{OSP}(1|2)$, generalizing the construction for the bosonic particle on

AdS₂. Gauging an SO(1,1) subgroup of OSP(1|2), we considered the action given by the square of the left current projected onto the bosonic part of the coset. For the massive case, we represented the mass-shell phase space as a coadjoint orbit of a timelike element of $\mathfrak{osp}(1|2)$, giving a well defined symplectic structure and a realization of the OSP(1|2) symmetry as the Poisson bracket algebra of the Noether charges. Our parametrization immediately yielded a description in terms of one bosonic and one fermionic oscillator and their canonical quantization led to a Holstein-Primakoff type realization of $\mathfrak{osp}(1|2)$.

Repeating the analysis for the massless case, we observed the decoupling of one fermion, which is an explicit demonstration of κ -symmetry in our setting. As this leaves only one real fermionic field, quantization of this system appears inconsistent.

There are a number of natural generalizations that would be of interest to explore in the future. One immediate open question is the quantization of the massless superparticle in AdS₂. The obstructions encountered due to the κ -symmetry suggests that one ought to consider the larger group SU(1,1|1), gauging an SO(1,1) \times U(1) subgroup, as considered in, for example, [32]. One could also consider an alternative \mathbb{Z}_4 grading for which an SO(1,1) subgroup of SU(1,1|1) is gauged. The resulting model describes a superparticle on AdS₂ \times S¹ and has been subject of the works [42] relevant for the Kerr/CFT correspondence [43]. Taking account of the κ -symmetries, the massless case of this model with fixed angular momentum on the S¹ should be classically equivalent to the massive model considered in this paper.

Furthermore, the theory considered in this paper can be understood as a truncation of various supercoset models related to known critical superstring backgrounds [39]. These include the supercoset PSU(1,1|2)/SO(1,1) \times SO(2), related to the AdS₂ \times S²(\times T⁶) string background [35,36], and D(2,1; α)/SO(1,1) \times SO(2) \times SO(2), related to AdS₂ \times S² \times S²(\times T⁴). Generalizing further to these cases may help in understanding the connection to the full critical superstring theory. In addition, in the first case it would be interesting to understand the relation to the construction of [37].

The extension to higher dimensional Anti de Sitter spaces is an important next step. The case of AdS₃ could be a helpful stepping stone in this direction as in the minimal case the isometry group of the superparticle action takes direct product form, OSP(1|2) \times OSP(1|2). Hence the results of this paper relating to the supergroup OSP(1|2) will be applicable therefor.

Finally, let us conclude by recalling that one of the eventual aims of this program is the application to AdS superstring theories, of interest in the context of the AdS/CFT correspondence, and the quantization of strings on these backgrounds from first principles.

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